ABSTRACT

The determination of accurate positions of stars is an essential task to obtain a reference coordinate system for precise navigation of ships, airplanes, satellites and space crafts. Stellar positions and proper motions will also help to investigate the structure and evolution of matter in the universe. From astronomical observations systematic errors have been reported by different authors using different techniques to detect and measure positions of point-light sources from digital images. An overview of this problem of image processing is given. The relative subpixel deviation will be defined and described. It provides interesting characteristics independent from the method of position retrieval or point spread introduced by any image deterioration. The relative subpixel deviation defines the lower limit of the achievable precision of positions. The function is the error correction term by itself, which yields a new concept of the error correction. A concept of a new simulation software is presented to further investigate the relative subpixel deviation. Features of the core framework are: support of different optical systems, sources of noise, and detectors types, like monochrome or color CCD and CMOS imagers, their pixel geometry, gaps and varying subpixel sensitivity functions. First results from the simulator software as well as results from new astronomical observations are presented as a proof of concept of the proposed approach to improve the current limit of position accuracy in the order of a few 1/100 of a pixel.

KEYWORDS

Image analysis, position detection, astronomy, astrometry.

1. INTRODUCTION

Astrometry denotes a special part of astronomy science which yields positions and proper motions of celestial bodies like stars, clusters of stars (e.g. open clusters, globular clusters), galaxies and distant quasars. Astronomy is interested in questions like how to gain an idea of the spatial distribution and move of matter in the universe. Taking precise positions from stars is also an essential task with a long tradition and of high cultural, social and economic importance. The stars span coordinates used as a precise reference coordinate systems for modern navigation of satellites and space crafts, airplanes, ships, vehicles and individuals on ground and also defines the fundamentals of time and calendar. With silicon imagers this is a modern task of digital image analysis. The complexity of astrometry with such detectors has been summarized by Monet (1988) and gives a repeated state of the art with his later introduction into CCD astrometry (Monet, 1992).

From the observers viewpoint distant stars generally present themselves as point-light sources with no dimension in x and y. Within the focal plane of any telescope optics, the image of a star will be deteriorated by the complete optical system and detector. The shape of the intensity function is called a point-spread function, hereafter abbreviated PSF, which is caused by several reasons. One major question is: How to retrieve the x/y coordinate of the blurred star image? From an intuitive approach this problem is solved by calculation of a centroid or estimate of the position by a fit with an assumed shape function. Obviously, it is possible to retrieve back x/y coordinate pairs within sub-pixel range by use of any of these techniques.
Of course, solutions to the problem exist based on pattern recognition and data retrieval. One of the first automated tools for image analysis and data extraction, DAOPHOT, was presented by Stetson (1987). The tool is well known, referred by many authors and part of the two major astronomical standard image processing packages: MIDAS (Banse et al. 1983) and IRAF (Tody, 1986). The algorithm presented with DAOPHOT originally was thought as a tool for photometric data extraction to measure intensities of stars in crowded fields. It uses pattern recognition for the star identification and tries to fit a model point-spread function (PSF). Where necessary, DAOPHOT tries a multiple fit where it finds crowded stellar fields containing close binary or multiple stellar images. A fit is suited to extract both, photometric information and x/y coordinate pairs from the image. Some observers use tools like DAOPHOT to obtain astrometric parameters (x/y coordinates), too. A major problem identified in the literature is the following question: How to retrieve the individual, spatially variant PSFs from the image taken with any observation? A lot of effort is seen from literature involved in the problem.

Maybe, the primary question should be formulated in a slightly different way: which precision may we expect from measures of a star position using digital imagers? Is there a possible limit?

2. DEVELOPMENT OF A NEW APPROACH

2.1 The role of the point-spread function

The point-spread function (PSF) in general is defined by the whole optical system and will deteriorate the image of a point-light source by some wide spread. This is recorded and digitized by the detector. According to Born & Wolf (1953) there are several distortion effects involved and caused by the optics itself. Examples are optical distortion of the whole image coordinate system, and local effects, like astigmatism or coma. The optics forms a band-width limited detector as seen from information theory. The star image is defined by a convolution of a delta function by the optical transfer function from the optical aperture and affected by different phase errors when light passes the optical system. Optical designers try to minimize these aberrations from the theoretical diffraction pattern, while maximizing the usable image area. However, it is impossible to create a perfect optics. Sampling with an digital detector means to fulfill the Whittaker-Shannon theorem for the measured diffraction pattern. This is the case with large space telescopes.

![Figure 1: An optical system forms a bandwidth limiting system. The image of a point-light source, like stars with large (infinite) distances, are given by the convolution of a delta function with the Fourier transform of the aperture of the optics.](image)

From ground-based observations the optical system not only designates the camera or telescope optics itself. Point-spread may also include effects caused by atmospheric turbulence which introduces further image deterioration. Additionally, the telescope may move during long-exposure time. This may result in tracking errors like scattered or curved, elongated, rotated patterns of the star images or multiple images. If we compare two different images taken, obviously we shall expect to find different wide-spread and thus a different PSF with every image. See Figure 2 for samples. Local variations of the PSF is introduced by
optical aberrations mentioned above. Starting from a certain diameter of the optical aperture and with long-exposure imaging, atmospheric point-spread will dominate at a larger scale compared to the size of the diffraction pattern. However, scale is still a very small angular resolution. The dimension of the squared pixel shown in the Figure 3 represents about 5 microns at the camera sensor which represents about 1 arc second (1/3600 of a degree) of the angular field of view. Although scale already seems to represent very small structures, the objective is to obtain stellar positions precisely and better than 1/100 of the pixel dimension to define a better angular resolution for the star coordinates.

Figure 2: Three different samples of time-sampled imaging of the same star cluster taken with an astronomical telescope at small focal length. a) Nearly well sampled image within the focal plane. b) A telescope tracking error introduced elongated or multiple images. c) Image taken with intentionally defocussed telescope optics.

Figure 3: Correct sampling in the case of ground-based telescopes: starting from a certain diameter of the optical aperture, atmospheric point-spread will dominate. The optics is adapted in such a way, that sampling does not refer to the small scale optical diffraction pattern, but the expected (mean) value of the point-spread caused by the atmosphere. Therefore, with some telescope designs and because of the large dynamic range large scale diffraction patterns can be found in addition, like the optical diffraction of a secondary mirror mount of the telescope. The PSF shows a typical cross-haired structure as a result from the brightest stars.

2.2 Discretization effects
Modern imaging detectors produce images built of pixels. The light passes small transparent apertures where the silicon is sensitive to convert light into a photo current which then will be digitized. These small apertures are the sensitive detector pixels. The sum of pixel coordinates found with the pixels given from a star image weighed by the pixel intensity may represent a good estimate for a x/y coordinate with sub-pixel precision. This is a calculation of a simple centroid of the measured stellar image. However, it has been shown, that it is more precise to have an idea of the PSF fitted with the data. A fit of the PSF can be achieved using a maximum-likelihood approach or another method of optimization to obtain x/y coordinates.

\[ i_{\text{pix}} = \int i(x,y) \, dx \, dy \]

**Figure 4:** The PSF and the detection process. A pixel of the detector spatially integrate the light defined by the local PSF (left). The result will be the discrete intensity distribution of the point-light source (right).

A main topic of discussion is the retrieval of a „good PSF“ to fit with. The question was answered earlier. Moffat (1969) presented a theoretical model of stellar profiles distorted by statistical effects of the earth atmosphere, and found with photographic film emulsions. This was the typical case before digital electronic CCDs started a new revolution in astronomy science. A later version of DAOPHOT presented by Stetson et al. (1990) provided different PSF models: Gaussian profiles, and Moffat functions. The ongoing discussion now outlines several problems suggested from the standard software DAOPHOT. Supposed error sources and problems with the determination of the true PSF has been described by Anderson & King (2000), Mighell (2005), Stubbs & Tonry (2006). A few authors presenting their own software solution with a different tool like DoPHOT invented by Schechter, P. L. et al. (1993). Mighell (2005) described a different approach and implemented a tool which uses discrete PSFs, called MATPHOT. The problems mainly are discussed to rely on the physical constraints of the image formation process. All authors in common claim, the accuracy of position measurement will improve, if an accurate PSF can be obtained. However, we may ask, whether this is the only solution for the problem.

From the ongoing discussion at least two noticeable papers are identified, where authors clearly identified and showed systematic deviations obtained from the data retrieval with tools and methods defined so far. Guseva (1995) applied CCD observations with an astograph, a special type of telescope optics designed for astrometric purpose. To show the position error several images have been recorded with small shifts of the star field on the CCD. Systematic deviations of the relative position measurements were found as a sinuoidal graph with an amplitude in the order of 0.05 pixel (Figure 5). A further systematic intensity deviation is noted to be in the order of 0.15 [mag]. The unit [mag] is a logarithmic unit according to Pogsons formula: \[ M = -2.5 \log_{10}(i) \], where \( i \) denotes the measured star intensity and \( M \) the logarithmic stellar magnitude [mag].

Anderson and King (2000) later presented similar x/y deviations within subpixel range obtained with observations from the Hubble space telescope. Although the telescope setup was completely different, as a small ground based telescope is compared to a huge space telescope working in a slightly undersampled manner, the trends surprisingly have been found in the same order of the pixel deviation compared to the work of Guseva (1995). Based on their data Anderson & King (2000) developed their own method to retrieve an analytical PSF from the data itself. The technique is shown to eliminate systematic errors and has been applied to images of star clusters taken with the Hubble space telescope. Anderson & King (2000) acknowledge their method will work reasonable only for large data sets with many stars contained in the images. Anderson & King (2006) later showed further consequences of their approach. One can see the effort...
to achieve and store any spatially variant PSF from the data. They also discussed variations from PSFs obtained, which is suggested to be caused by instabilities of the Hubble space telescope focus and guiding. *Anderson et al. (2006)* tried to apply their method with images taken from ground-based observations. Here a different situation is presented. Problems now result from much larger variations of a PSF disturbed by statistical effects of the earth atmosphere. They also describe constraints and relationships between an obtained PSF and optical aberrations of the different telescope they used for new observations.

![Figure 5: The effect of the pixel structure on the position error in subpixel range according to Guseva (1995). The statistical error presented like a sine curve with an error amplitude of about 0.05 pixel for a special CCD camera setup.](image)

The primary work of *Anderson & King (2000)* may lead to several other consequences about possible interpretations and verifying their conclusion. At least the article represents a further documented case, where it was tried to fit different PSFs with different approaches and suggested uncertainties to a set of astronomical images. They also used the software DAOPHOT (unfortunately, they did not describe details). As the authors reported, any of their first approaches did not lead to completely wrong stellar positions. Instead they reported similar systematics with the deviation functions, like the one found by *Guseva (1995)*. With the several different approaches, the error was found in the order of 0.02 pixel for the Hubble telescope. This order of an error again seems to define some limit of precision for the position estimate. A further conclusion may be: Several different software methods used so far, will produce similar results. This may mean different fitting approaches provide a relatively stable and robust method to obtain positions with different telescope setups. In contrast, authors claim such methods not being able to obtain precise positions. This contradictory may mean some advantage, but the limits have to be verified.

Of course, a precise model of a real PSF requires to know its structure within subpixel range (which will mean, the problem remains the same for a small amount of data). Which method will yield a „correct“ position? Techniques to fit a function into a set of data may represent a better approach than simply taking a mean value from the data, like a simple centroid. However, for statistical reasons, even the fit may fail or represent a false estimate, if there exists no plausible relation between an assumed function and the data. For statistical reasons, we shall not assume to have found a better position obtaining any function with any precision, extracted from and fitted back into the data, regardless of the effort involved to do so.

According to *Brunner et al. (2001)*, astronomy produces large sets of data. Costs of any approach in the sense of effort, time and resources (human and technical) need to be calculated to compare the effectiveness of any method. If the result shall be a precise position of a star one may ask about the importance of having found an accurate PSF with any precision. This means, it might be of less importance, whether having calculated a worse PSF or a good PSF. There exists, however, a relationship between an observed coordinate O compared to an expected (or computed) coordinate C within any reference coordinate system which is chosen. The relation between O and C represents a systematic error deviation relation O-C. The order and shape of the error deviation is given by the quality of data, the detector, camera and the method.

### 2.3 Characteristics of the relative subpixel deviation function (RSDF)
With an ideal detector, the small (pixel) areas sensitive to light have a finite (not necessarily rectangular) spatial dimension. The pixel are regularly distributed as a rectangular image and shall have a uniform sensitivity to the light. The pixel values at positions \(p_1, p_2, \ldots, p_n\) contained within the image shall be numbered and ordered by their coordinates. If one finds any deviation between an expected and an observed image in the local region of the closest pixel with any method, this is called the relative subpixel deviation, hereafter called RSD. The RSD defined below has several useful characteristics. It will have these characteristics without any knowledge about the PSF, the object structure, or the technique used for position retrieval, as there will be no assumptions with it.

The vector \(C_n\) shall be the true position of an object relative to the coordinates \(p_n\) of the closest pixel in the image with vector components \(p^k\leq C^k\) (which are the coordinates of the pixel and the true position). The RSD \(f_{RSD}\) now is defined as:

\[
f_{RSD}(C_n, P, M) = O_n - C_n,
\]

with the observed position \(O_n\) (e.g. measured position of the star), the true position of the star \(C_n\), the point spread \(P\) of the image and the method of position retrieval \(M\). Without loss of generality the two parameters \(P\) and \(M\) shall be taken as non-variant parameters. Thus, with a (mean) constant PSF and a certain method of position retrieval Eq.1 yields a simple function \(f_{RSD}(C_n)\) of the true position of the star \(C_n\), which shall be denoted by the shorthand term \(f_{RSD}\). Eq.1 yields the following characteristics of the RSD \(f_{RSD}:

1.) The RSD is defined within \([0,1]\) having values within the interval \([-1,1]\) where ever it is defined.
2.) The finite integral \(\int_{0}^{1} f_{RSD}(x)^2 \, dx\) is the lower limit of the expected standard deviation \(\sigma^2\) of \(O_n\).
3.) The RSD itself defines the correction term.

Proof: While \(C_n\) defines the true position, the observed position is \(O_n\). If the object image at position \(O_n\) is shifted across the detector, the detector will record the same intensity distribution at position \(O_{n+1}\). By definition, we find its true position now at \(C_{n+1}\). This is nothing else, than a shift in one dimension by 1. Any shift between \(O_n\) and \(O_{n+1}\) takes place within a fractional subpixel range of \([0,1]\). It follows: the RSD value is defined only in the interval \(f_{RSD} \in [-1,1]\), which is an upper limit for a subpixel deviation given by the RSD for any shift between \(O_n\) and \(O_{n+k}\) with index \(k\), which is a natural number. Obviously we have found a relation which uses similarity within a well defined neighborhood where the RSD values will yield a relationship between observed and real positions. This is nothing else, than the ability to compare the positions of objects found in the local neighborhood of the image. Obviously it is possible to use the RSD as a subtractive correction term for observed x/y coordinates within a certain region of the image, if not the whole image. The observed coordinates are displaced from the real position where we expect the image. The true position \(C_n\) now can be obtained by the equation:

\[
C_n = O_n - f_{RSD}
\]

Eq. 2

This is true for the one-dimensional case. For the n-dimensional case any \(O_n\) forms a vector of \(n\) dimensions. It can easily be shown, this is also true for the single components of the n-dimensional vectors \(O_n\) and \(O_{n+k}\) or any combination of the vector components. The geometric interpretation in two dimensions is simply an integer shift along a row or a column of the image, or both. The pixel coordinate system of the image itself defines a fixed reference coordinate system, which enabled the RSD to describe subpixel deviations of the position error. In practice a centered RSD, which is pixel centered, may be also used. The centered RSD may observe the shift in the interval \([-0.5,0.5]\) from the center of a pixel \(p_n\). The centered RSD is nothing else, than the RSD itself, because the coordinate system is shifted by 0.5 pixel and the interval \([0,1]\) shifted by -0.5 pixel. Fig. 6 will represent such a pixel centered RSD.

Further interpretations follow: The RSD defines any limit of precision for obtaining x/y positions with any given method and telescope imaging system (which surely might work with any optical system, too). In general we expect their values to have small amplitudes for good fitting methods. For a set of measures the standard deviation \(\sigma\) of the error between the true position \(C_n\) and the observed \(O_n\) is given by Equation 3:

\[
\sigma^2 = 1/(N-1) \sum (O_n-C_n)^2 > 1/N \sum (O_n-C_n)^2
\]

Eq. 3

because of the factors of the sum. With the interval \([0,1]\) and the limes \(N\rightarrow\infty\) the right sum of Eq.4 yields the integral \(\int f_{RSD}(x)^2 \, dx\). As a result, the integral is smaller than the expected \(\sigma^2\) for any large value \(N\),
which is the number of measures. With the square root as a monotone function, the relationship found in Eq. 3 is still valid. Therefore, the square root of the integral of \( f_{\text{RSD}} \) defines the lower limit to the expected value of the standard deviation \( \sigma \).

### 2.4 Requirements for a simulation software

The shape of the RSD and the lower limit defined by the RSD will depend on the properties of a real detector, observational parameters and the method of the position detection. Thus, it will be comfortable to find a method to predict and study the behavior of the RSD. With the complexity of the parameters mentioned above, a simulator will be a reliable tool to predict position deviations and minimize the errors for any given telescopic camera system, any given PSF and any method of position detection. This shall be proven by further studies of the RSD.

An object-oriented approach makes it easier to create such a tool and to understand how it works. The collaborators (objects) within the program are involved in the physical image formation process. Regardless of the real detector and point-spread, the collected light is spatially integrated and represented by a digital value. A real CCD or CMOS detector will have a certain pixel geometry with gaps between pixels, spatially varying pixel dimensions and other constraints. According to Piterman & Ninkov (2002), spatially variant subpixel response to illumination has to be assumed even with light falling onto a single pixel. Further development of Estribeau & Mangan (2005) predicted pixel cross-talk of detected intensity values found from CMOS detectors. From carefully reading their work, it is not quite clear, whether they again described similar effects like the discretization effect documented by Guseva (1995). However, the existence of such effects shall be assumed. Several electronic effects from the amplifier and discretization may further deteriorate the digitized signal. One possible deterioration effect of the discretization stage is seen as a comb in the histogram. The effect is caused by a varying probability to find discrete numbers depending on the intensity rate and similar effects caused by bit errors introduced with an analog digital converter. It is not necessarily important for the proposal of this approach to count all the effects. However, such effects shall kept in mind while modeling the real component classes either theoretically or obtained from lab calibration of a real detector.

Behavior and methods of the detector class can be identified without any knowledge about all effects. The collaborators (objects) within the program are involved in the physical image formation process. Regardless of the real detector and point-spread, the collected light is spatially integrated and represented by a digital value. A real CCD or CMOS detector will have a certain pixel geometry with gaps between pixels, spatially varying pixel dimensions and other constraints. According to Piterman & Ninkov (2002), spatially variant subpixel response to illumination has to be assumed even with light falling onto a single pixel. Further development of Estribeau & Mangan (2005) predicted pixel cross-talk of detected intensity values found from CMOS detectors. From carefully reading their work, it is not quite clear, whether they again described similar effects like the discretization effect documented by Guseva (1995). However, the existence of such effects shall be assumed. Several electronic effects from the amplifier and discretization may further deteriorate the digitized signal. One possible deterioration effect of the discretization stage is seen as a comb in the histogram. The effect is caused by a varying probability to find discrete numbers depending on the intensity rate and similar effects caused by bit errors introduced with an analog digital converter. It is not necessarily important for the proposal of this approach to count all the effects. However, such effects shall kept in mind while modeling the real component classes either theoretically or obtained from lab calibration of a real detector.

The author implemented a preliminary version of the simulation core framework with the Java programming language. The following question shall be answered: How does the RSD look like, if one tries to obtain x/y coordinates of an assumed Gaussian intensity profile with an ideal, but noisy detector and using a simple Gaussian fit to retrieve back the measured displacement of the star from the image?

Implementation details are a consequence of the detector model and the parameters from astronomical observation practice. Details about practice in astronomical imaging with digital detectors are found with Howell (2006) or Berry & Burnell (2006). To answer the question above a well sampled image shall be
assumed as follows. The Gaussian stellar profile is set to 2 pixel full width at half maximum to fulfill the Whitaker-Shannon sampling theorem. This intensity distribution is integrated spatially for every pixel in the image. A Newton integral approximation now obtained pixel intensities. Real CCD detectors used within astronomy currently have a physical full well capacity of about 100,000 electrons (e⁻) per pixel with some conversion rate between photons and electrons in the silicon. The measured pixel values usually are digitized using 16 Bit analog-digital converters. Like astronomers would expose an image, the digital number range is chosen to values of about 75% not to find the brightest star saturated on the detector. This yields a peak of the Gaussian at about 45,000 digital units and a Gaussian noise for every pixel. The synthetic stellar images now contain integer values within this value range. The images have been fitted with a continous Gaussian distribution having the same values of FWHM and peak intensity. The position fit was applied using a least-square approach with the downhill simplex optimization method in two dimensions (Press et al., 2005). All calculations have been done with double precision. Obviously a „false“ PSF is assumed, because values of such a PSF are spatially integrated values of the Gaussian, not discrete values of the Gaussian itself. However, this deviation is chosen by intention to show the typical effect of the displacement with a falsely assumed PSF which is typical for most software packages.

Figure 6 shows the resulting plot for the RSD for x-coordinate obtained from 1000 random stars plotted against the true position in the interval of [-0.5,+0.5] pixel (x-axis). A simple fit with a polynomial function of 4th order was applied to the plot of the values. The polynomial fit will not take into account, that a further boundary condition says, values at deviation +0.5 and -0.5 are identical. A set of periodic functions obviously will define a much better approach to fit with the boundary conditions of the RSD. The plot not only shows scattered positions, but also the systematic trend of the RSD reported earlier. A correction of the systematic error introduced by the RSD is obtained by subtraction of the polynomial function from the measured positions.

As shown by theory, the integral \( \int_{-0.5}^{+0.5} [f_{\text{RSD}}(x)]^2 \, dx \) will define a lower limit of the achievable standard deviation and thus the best achievable position error from the measured values. From the simulation the integral of the obtained RSD obviously will not end up with a zero value, but instead present some positive value as a lower limit of achievable precision. For the demonstration of the effect a time-series of 70 images of the open stellar cluster NGC 6819 has been taken with a Vixen VC200L telescope, which is a derived Cassegrain telescope design optimized for photographic applications. A Canon EOS 40D digital SLR camera was used to record the image series with exposure time of 30 sec. The subsequent images have been taken within one hour of observation and stored digitally in Canons TIFF/EXIF raw format (Bauer, 2007).
The position error is expected to depend on the signal-to-noise ratio of the stars. In a first step all stars with intensities above $10\sigma$ of image noise have been extracted to evaluate their positions by a Gaussian fit. Again, a least-square approach with the downhill simplex optimization method was applied to fit the positions. The full width at half maximum of the Gaussian was estimated from visual inspection of the images. In a second step the mean shift between two sequentially recorded images was calculated from all stars identified in both images. A maximum shift in the order of a few pixel is found between the single images (introduced due to technical instabilities of the telescope mount and effects of the earth atmosphere). For every star the standard deviation of the position was calculated from all images where the star could be identified. Figure 8 represents a double logarithmic scale for both, the intensity and standard deviation. Obviously, the brightest stars (left) show a constant scatter limit, while fainter stars (right) will show the expected dependency of the standard deviation from intensity. A few close stars and cosmic events detected have been mistakenly identified by the automatic detection process across all images with large errors in the order of a pixel. No RSD correction has been applied. The lower limit of the precision found with the RSD (dotted line) is estimated at about a logarithmic value of -1.4 of the standard deviation which means a limit in the position error of about 0.04 pixel ($\pm$ 0.015).

Figure 7: The open stellar cluster NGC 6819 was chosen as a test candidate to verify the theory of the RSD. The image shows the inner 50 percent of the whole field of view obtained with an VC200L telescope and a Canon EOS 40D. Stars detected within the field of view span a dynamic range of 7 stellar magnitudes or about 1:1000. The result shown is a re-centered composite obtained from 70 exposures with the shift-and-add method using subpixel precision. The original colored image is inverted and reduced to grayscale for the print reproduction.
Figure 8: The effect of the RSD limit on real measures of star positions of NGC 6819. The RSD forms a constant bias to the standard deviation even for the brightest stars shown in the left side of the plot. Not any amount of data will gain a further improvement on the position error until the systematic error introduced by the RSD is corrected or any other method improves accuracy. The plot represents data from one single color channel of the RGB camera.

3. CRITICAL EVALUATION

The RSD has some interesting characteristics from theory. These characteristics do not depend on any knowledge about the true PSF, nor are they influenced by techniques used to derive the positions. However an evaluated RSD is valid only for a certain PSF and method. It is supposed that this will also include a mean value of a statistical varying PSF. A simulation framework was proposed and implemented to obtain and explore such RSDs from theory and/or real lab measures with any imaging detectors system. It is shown from theory and demonstrated with real measures, that the integral \[ \int f_{RSD}(x)^2 \, dx \] defines a lower limit for the standard deviation found with position retrieval. The RSD is a systematic residual from the detection of light with a discrete detector which integrates the light spatially onto a single pixel intensity. One way to eliminate this effect is the determination of a valid PSF model. However it has been shown by different authors that this will mean a complicate workflow and will work reasonable only with a large amount of data. This technique of evaluating a PSF from the data also leaves the open question about plausibility of the reconstructed PSF and the conclusion to have found a “better” position with it. On the other hand, the RSD defines the correction term by itself to improve position accuracy by subtracting this function from the data measured. Thus, it is expected to improve position accuracy by factors using the RSD and without any further knowledge about the true point spread. With the true PSF unknown, a simulation of the detector with observational parameters will help to understand the behavior of the RSD in general. The solution for all these cases now can be a model of the RSD taken from observations or simulations of the optical image formation process. This will mean an overall improvement of position accuracy in general with this technique. This shall be proven by future investigations of the RSD with special cases commonly found in astronomical imaging. With a huge amount of data, the RSD may be retrieved from the data, too. A spatially variant PSF shall be assumed in any case, as the work of Anderson & King (2000) implies. This will depend on the optical system chosen. The general concept of the RSD might be useful as a general solution even in this case. Within a local portion of the image, the concept of the RSD is still true as it only requires and uses similarity of features, but might be restricted to a certain portion of the image with an assumed constant (mean) PSF for this portion. The influence of a varying PSF on the RSD should be investigated where applicable. Finally, one major advantage of the application of the RSD over techniques published so far
might be an easy application to and improve of such position data already obtained with any feature extraction software without the need of a completely new image reduction.

4. CONCLUSION

This paper presented a brief discussion of techniques used to compute positions of point light sources in astronomy science. In contrast to the ongoing discussion in literature, a different approach is proposed to correct the computed positions of point-light sources by a simple correction term called the relative subpixel deviation function RSD. Of course, optical distortion of the image coordinate system has to be corrected in order to be able to apply this linear approach. Again, the RSD may help to determine such relationships.

In the near future a few more questions are of importance. What RSD is achieved with typical telescope setup in astronomy? From first results of the simulation trends are seen for typical oversampled and undersampled cases of imaging. The latter (undersampling) is the typical case with the current Hubble space telescope design. Contradictory some authors proposed oversampling for better positions accuracy (which means extended exposure times or a decrease of the S/N). Typical point spread is found in the range of a diffraction pattern to Gaussian smoothing (oversampled) at large focal lengths. A systematic analysis with the current simulator framework shall answer open questions. A major question within the context of the current project is how to improve super-resolution imaging in astronomy. The method of correcting for the RSD is expected improve the development and application of super-resolution techniques with long-exposure imaging in astronomy. High-resolution imaging within subpixel range with any image deterioration will be a powerful technique to improve existing algorithms for the data extraction of photometry and astrometry.

At least, a new convenient method has been found to improve the quality of astrometric measures of point light sources without the requirement of knowledge about the point spread. An improved position accuracy at lowered costs will mean an advantage over existing techniques which currently require large data sets in astronomy science. As astronomy usually produces a large amount of data, this may mean an improvement of the overall performance of astrometric data analysis. This work is part of a new project to develop improved techniques for astrometric and photometric data analysis for digital imaging within the context of astronomy.

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